MAT 121 §3.5 I #s 22, 33, 38, 47, 50, 53, 55, 68, 71

#s 21–32 Tell the max # of zeros each polynomial can have. Use Descartes' Rule of signs to determine how many positive & negative zeros each polynomial function may have. DO NOT ATTEMPT TO FIND THE ZEROS.

(22) $f(x) = 5x^4 + 2x^2 - 6x - 5$ ONE Positive.

$f(-x) = 5x^4 + 2x^2 + 6x - 5$ One Negative

Max # of zeros = Degree = 4

#s 33–44 List the potential rational zeros of each polynomial function DO NOT ATTEMPT TO FIND THEM!

(33) $f(x) = 3x^4 - 3x^3 + x^2 - x + 1$ $\pm \dfrac{\text{FACTORS OF } a_0}{\text{FACTORS OF } a_n}$

$a_n = 3,\ a_0 = 1$: $\boxed{\pm 1, \pm \frac{1}{3}}$

(38) $f(x) = 6x^4 - x^2 + 2$

$a_n = 6,\ a_0 = 2$

$\boxed{\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 2, \pm \frac{2}{3}}$

#s 45–56 – Use Descartes' Rule of Signs & Rational Zeros Theorem to find all the real zeros of each polynomial function. Use the zeros to factor f over the real numbers.

MAT 121 §3.5 I #s 47, 50, 53, 55, 68, 71

47 $f(x) = 2x^3 - x^2 + 2x - 1$ 3 or 1 positive

$f(-x) = -2x^3 - x^2 - 2x - 1$ No Negative zeros

$a_n = 2,\ a_0 = -1$: $\pm 1, \pm \frac{1}{2}$ Ditch the negatives

$+1, +\frac{1}{2}$ are our two candidates:

$$\begin{array}{r|rrrr} 1 & 2 & -1 & 2 & -1 \\ & & 2 & 1 & 3 \\ \hline & 2 & 1 & 3 & 2 \neq 0 \text{ No} \end{array}$$

$x - \frac{1}{2} \rightsquigarrow$

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & 2 & -1 \\ & & 1 & 0 & 1 \\ \hline & 2 & 0 & 2 & 0 = 0 \text{ Yes} \end{array}$$

$\leftarrow 2x^3 - x^2 + 2x - 1$

$2x^2 + 2 \rightsquigarrow$

$2x^2 + 2$ is irreducible over $\mathbb{R}$

So, $f(x) = \left(x - \frac{1}{2}\right)(2x^2 + 2)$
Real zeros: $x = \frac{1}{2}$

OR $(2x-1)(x^2+1)$ is O.K.

50 $f(x) = x^4 - 3x^2 - 4$ 1 positive real zero

$f(-x) = f(x)$ (EVEN) 1 negative real zero

$a_n = 1,\ a_0 = -4$: $\pm 1, \pm 2, \pm 4$ are candidates

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -3 & 0 & -4 \\ & & 1 & 1 & -2 & -2 \\ \hline & 1 & 1 & -2 & -2 & -6 \text{ No} \end{array}$$

NOTE 0's where there are missing terms in $f(x)$.

MAT 121 §3.5 I #s 50, 53, 55, 68, 71

50

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -3 & 0 & -4 \\ & & 2 & 4 & 2 & 4 \\ \hline & 1 & 2 & 1 & 2 & 0 \end{array} \quad \text{Yes.}$$

This says $f(x) = (x-2)(x^3+2x^2+x+2)$

We work now with x^3+2x^2+x+2, now that we've "split off" a factor of $x-2$.

Since $x=2$ uses up all our positive zeros, we start with $x=-1$, $x=-2$, ...

$$\begin{array}{r|rrrr} -1 & 1 & 2 & 1 & 2 \\ & & -1 & -1 & 0 \\ \hline & 1 & 1 & 0 & 2 \end{array} \quad \text{No}$$

6

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 1 & 2 \\ & & -2 & 0 & -2 \\ \hline & 1 & 0 & 1 & 0 \end{array} \quad \text{Yes}$$

So $x^3+2x^2+x+2 = (x+2)(x^2+1)$ and

so $\boxed{f(x) = (x-2)(x+2)(x^2+1)}$

$\boxed{\text{Zeros: } x = \pm 2}$

NOTE A student who knows factoring could factor $f(x)$ FAST by grouping

47 Factoring it with "old skills"

$$f(x) = 2x^3 - x^2 + 2x - 1$$

$$= x^2(2x-1) + 1(2x-1)$$

$$= (2x-1)(x^2+1)$$

50 with "old skills"

$$f(x) = x^4 - 3x^2 - 4$$

$$u^2 - 3u - 4$$

$$= (u-4)(u+1)$$

$$= (x^2-4)(x^2+1)$$

$$= (x-2)(x+2)(x^2+1)$$

You can't always count on "old skills," but it's nice when you can confirm your work with them. Old skills are fine for checking, but we test on Chapter 3 skills.

53 $f(x) = x^4 \underbrace{+ x^3 - 3x^2}_{1} \underbrace{- x + 2}_{2}$

2 or 0 positive zeros

$f(-x) = \underbrace{x^4 - x^3}_{1} \underbrace{- 3x^2 + x}_{2} + 2$

2 or 0 negative zeros.

Rational zeros: $\pm 1, \pm 2$

$a_n = 1, \ a_0 = 2$

OLD SKILLS NOT ENUFF on #53

(53)

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & -3 & -1 & 2 \\ & & 1 & 2 & -1 & -2 \\ \hline 1 & 1 & 2 & -1 & -2 & 0 \\ & & 1 & 3 & 2 & \\ \hline & 1 & 3 & 2 & 0 & \end{array}$$

Try $x=1$, again. It might have multiplicity $m=2$!

Yes (first division remainder 0)

Yes. It does!

Work so far says
$f(x) = (x-1)^2(x^2+3x+2)$ & we still need to break down x^2+3x+2, if possible.
Well, it IS possible: $x^2+3x+2 = (x+2)(x+1)$

So $f(x) = (x-1)^2(x+2)(x+1)$

zeros: $x = 1, -1, 2$
$x=1$ has multiplicity $m=2$.

(55) $f(x) = 4x^5 - 8x^4 - x + 2$ 2 or 0 pos. zeros

$f(-x) = -4x^5 - 8x^4 + x + 2$ 1 neg. zero

$a_n = 4$, $a_0 = 2$: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2,$

$$\begin{array}{r|rrrrrr} -1 & 4 & -8 & 0 & 0 & -1 & 2 \\ & & -4 & 12 & -12 & 12 & -11 \\ \hline & 4 & -12 & 12 & -12 & 11 & -9 \end{array}$$

No

55

-2	4	-8	0	0	-1	2
		-8	32	-64	128	
	4	-16	32	-64		No

$-\frac{1}{2}$	4	-8	0	0	-1	2
		-2	5	$-\frac{5}{2}$	$\frac{5}{4}$	$-\frac{1}{8}$
	4	-10	5	$-\frac{5}{2}$	$\frac{1}{4}$	No

$-\frac{1}{4}$	4	-8	0	0	-1	2
		-1	$\frac{9}{4}$	$-\frac{9}{8}$	$\frac{9}{32}$	Nah
	4	-9	$\frac{9}{4}$	$-\frac{9}{8}$	$-\frac{23}{32}$	

Try the "+" candidates:

1	4	-8	0	0	-1	2
		4	-4	-4	-4	Nah
	4	-4	-4	-4	-5	

2	4	-8	0	0	-1	2
		8	0	0	0	-2
	4	0	0	0	-1	0 Finally!

So $f(x) = (x-2)(4x^4-1)$

$$= (x-2)(2x^2-1)(2x^2+1)$$

$$= (x-2)(\sqrt{2}\,x-1)(\sqrt{2}\,x+1)(2x^2+1)$$

MAT 121 §3.5 I #s 55, 68, 71

55 So, the zeros are $x = 2, \pm \frac{1}{\sqrt{2}}$

We obtain the $\pm \frac{1}{\sqrt{2}}$ by solving

$\sqrt{2}x + 1 = 0$, $\sqrt{2}x - 1 = 0$

$\sqrt{2}x = -1$ $\sqrt{2}x = 1$

$x = -\frac{1}{\sqrt{2}}$ $x = \frac{1}{\sqrt{2}}$

#s 57-68 Solve Each Equation in the real number system.

68 $2x^4 + x^3 - 24x^2 + 20x + 16 = 0$

FOR THIS ONE, A GRAPHER CAN REEEALLY HELP!

My grapher suggests I should look at $x = -4, -\frac{1}{2}, 2,$ and $x = 2$ is likely of multiplicity 2.

Descartes': 2 or 0 positive zeros

$f(-x) = 2x^4 - x^3 - 24x^2 - 20x + 16$; 2 or 0 negative.

Rational zeros: $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 4, \pm 8, \pm 16,$

$$\begin{array}{r|rrrrr}
-4 & 2 & 1 & -24 & 20 & 16 \\
 & & -8 & 28 & -16 & -16 \\
\hline
2 & 2 & -7 & 4 & 4 & 0 \\
 & & 4 & -6 & -4 & \\
\hline
 & 2 & -3 & -2 & 0 &
\end{array}$$

$x = -4$ is a zero

Yes!

$x = 2$ is a zero

Yes!

So,

$2x^2 - 3x - 2 = \underbrace{(2x+1)}_{x=-\frac{1}{2}}\underbrace{(x-2)}_{x=2}$

$x \in \{-4, -\frac{1}{2}, 2\}$

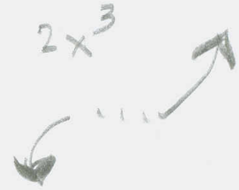
MAT 121 §3.5 I # 71

#s 69–80 : Find the intercepts. Graph
(Hint: Use factored form.)

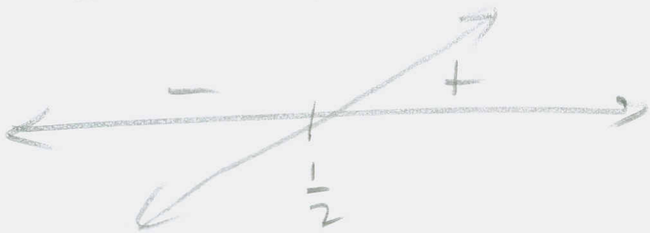
71 NOTE: we factored $2x^3 - x^2 + 2x - 1$ in #47

$47 - f(x) = 2x^3 - x^2 + 2x - 1$

E.B.: $2x^3$



$= (x - \frac{1}{2})(2x^2 + 2)$



#71 is sucky. But here goes:

Near $x = \frac{1}{2}$, it looks like

Without Calculus, it's hard to say much more about $f(x) = 2x^3 - x^2 + 2x - 1$